

Exploring the foundations of quantum mechanics using Monte Carlo simulations of the Freedman-Clauser experimental test of Bell's Inequality

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Monte Carlo simulations of the Freedman-Clauser experiment are used to test the generic wave function collapse model of Quantum Mechanics, a local realistic model, and a dynamical state reduction model of wave function collapse. The simulated results are compared to the actual results of the experiment which confirmed the quantum mechanical calculation for nine different relative angles between the two polarization analyzers. For each simulation 5×10^7 total simulated photon pairs were generated at each relative angle. The generic wave function collapse model closely followed the general shape of the theoretical calculation but differed from the calculated values by 2.5% to 3.3% for angles less than or equal to $\pi/8$ and differed by 15.0% to 52.5% for angles greater than or equal to $3\pi/8$. The local realistic model did not replicate the experimental results but was generally within 1% of a classical calculation for all analyzer angles. A dynamical state reduction/collapse model, approximated by using a “smeared” polarization, yielded values within 1% of the quantum mechanical calculation and provides an independent estimate of the correlation length used in these models of $r_c = (1.04 \pm 0.14) \times 10^{-5}$ cm.

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I. INTRODUCTION

Quantum mechanics (QM) is one of the most successful theories in the history of science. Since its development in the early twentieth century, the predictions of QM have been repeatedly confirmed by experiment to extremely high precision. The physical interpretation of what the mathematical formalism represents, however, has been the subject of much debate. Most physicists simply apply the formalism and are either not concerned with a physical interpretation or accept the Copenhagen interpretation as reasonable. Exactly what constitutes the Copenhagen interpretation, however, is not always completely clear. According to Faye [1] the Copenhagen interpretation is generally related to indeterminism, a statistical interpretation of the wave function, and Bohr's concept of complementarity. To most physicists, however, it also includes the collapse of the wave function during a measurement. This leads to the “measurement problem” which is generally defined as the conflict between the linear dynamics of QM and the non-linear collapse of the wave function during a measurement.

In their 1935 paper Einstein, Podolsky, and Rosen (EPR) [2] argued that QM was incomplete. In their analysis EPR distinguish between “physical reality” and the “physical concepts” of a theory that are intended to correspond to physical reality. EPR did not attempt to extensively define “physical reality” but rather asserted that - “If, without in any way disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity”.

While not specifically stated, EPR tacitly assume that “elements of physical reality” have precise values. The physical quantities predicted by a particular theory and measured by a particular experiment contain a level of uncertainty, but the underlying physical reality is assumed to be an exact quantity independent of any theory or measurement. This assumption is critical to their analysis for without it, no theory would be able to meet the above criterion which they believe is both a “reasonable” and “sufficient” way of “recognizing a physical reality”.

EPR also require locality and admit that their conclusion would not be valid if non-locality is allowed. However, they reject non-locality stating “no reasonable definition of reality could be expected to permit this”. Since QM precludes precise simultaneous determination of two physical quantities with non-commuting operators (e.g., position and momentum), EPR conclude QM must be incomplete.

Later authors attempted to resolve the EPR paradox by postulating so called “hidden variables” theories. J.S. Bell [3] made the issue more explicit and proved an inequality for what he termed “local realistic theories” that is violated by QM calculations for an entangled state of a composite system. Bell's work provided an experimental framework to resolve the apparent conflict between local realistic or hidden variables theories and QM.

A specific experiment to test Bell's inequality was proposed by Clauser, Horne, Shimony, and Holt [4] in 1969. The proposed experiment measured the correlation in linear polarization of two entangled photons emitted in a $J = 0 \rightarrow J = 1 \rightarrow J = 0$ atomic cascade of Calcium.

The proposed experiment was carried out by Freedman and Clauser in 1972 [5] for nine different relative angles between their polarization analyzers and confirmed, within the experimental error, the QM calculation at each angle. The results of Freedman and Clauser have since been confirmed by several other experiments [6, 7],

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(for complete review of experiments related to the Bell inequality see [8]).

The results of Bell inequality experiments have lead to a general acceptance of the non-local nature of QM, but have not resolved the measurement problem or the mechanism of wave function collapse. One avenue of research addressing this issue is Collapse Theory (dynamical reduction models) which attempts to modify the dynamical equations of QM to address the transition from microscopic to macroscopic objects [9, 10] (for a review of dynamical reduction models see [11] and the most recent progress on a relativistic state reduction model see [12]). As yet no definitive experimental test of these models has been performed.

It is important to note that the QM calculation tested by the Freedman-Clauser experiment and others does not require physical reality to have precise values nor does it require non-locality. The QM calculation simply begins with an assumed initial state of the system and applies the mathematical techniques set forth in QM to determine the probabilities associated with the possible final states that are expected to occur in the experiment. It is the physical interpretation of QM that leads to non-locality and a generally accepted concept of physical reality that leads to precise values of physical quantities.

The focus of this analysis is to evaluate, using Monte Carlo simulations of the Freedman-Clauser experiment, the generic non-local collapse of the wave function interpretation of QM, a local realistic interpretation, and an alternative physical interpretation wherein physical reality is not assumed to include precise values for physical quantities. The latter interpretation is intended to simulate dynamical state reduction/collapse models where physical quantities are “smeared” using Bedingham’s terminology [12].

Dynamical state reduction/collapse models add a stochastic term into the dynamical equation of QM corresponding to the reduction process (for a review of wave-function collapse models see [13]). This process is “formally identical to an approximate position measurement” [9]. Early work on dynamical state reduction/collapse models introduced the concept in this way [14, 15]. In his relativistic state reduction model Bedingham [12] utilizes a mediating field (called a “pointer field”) to “smear” the interactions.

To approximate dynamical state reduction/collapse models we therefore evaluate a model where the photon polarization is not assumed to have a precise value but rather is random with a Gaussian distribution about an appropriate mean.

We first utilize the generic wave function collapse interpretation of QM, with it’s inherently non-local characteristics, as the physical concept and demonstrate it provides a close, but not precise, match to the QM calculation.

We then show that a “local realistic” physical concept is completely inconsistent with the QM calculation and the results of experiment, but rather is in close agreement

with a classical calculation of the expected coincidence rate.

We then remove the assumption that physical reality contains precise values for physical quantities and evaluate the dynamical state reduction/collapse model approach and find that it more accurately corresponds to the predictions of QM that were confirmed by the Freedman-Clauser experiment and others.

II. QM CALCULATION OF THE COINCIDENCE RATE

Following the procedure in Horne [16] the probability of transmitting a linearly polarized photon using a real analyzer with known efficiencies is given by the following efficiency matrix

$$P = \begin{pmatrix} \epsilon_{\parallel} & 0 \\ 0 & \epsilon_{\perp} \end{pmatrix}$$

where ϵ_{\parallel} is the probability of transmitting a photon with linear polarization parallel to the analyzer and ϵ_{\perp} is the probability of transmitting a photon polarized perpendicular to the analyzer (leakage). Transforming to a basis for a photon with an arbitrary angle of polarization, ϕ , the probability of transmission for both QM and classical calculations becomes

$$P(\phi) = \epsilon_{\parallel} \cos^2 \phi + \epsilon_{\perp} \sin^2 \phi \quad (1)$$

The expected coincidence rate, R_{ϕ}/R_0 , in the Freedman-Clauser experiment [4, 16] predicted by QM is given by Equation 2.

$$R_{\phi}/R_0 = \frac{1}{4}(\epsilon_{\parallel}^1 + \epsilon_{\perp}^1)(\epsilon_{\parallel}^2 + \epsilon_{\perp}^2) + \frac{1}{4}(\epsilon_{\parallel}^1 - \epsilon_{\perp}^1)(\epsilon_{\parallel}^2 - \epsilon_{\perp}^2)F_1(\theta) \cos 2\phi \quad (2)$$

where the superscripts refer to the two different analyzers; $F_1(\theta)$ is a function of the acceptance angle of the detectors; R_{ϕ} is the measured coincidence rate with the analyzers at a relative angle ϕ ; and R_0 is the measured coincidence rate with both of the analyzers removed. R_{ϕ}/R_0 therefore measures the effective coincidence rate and removes any effect of the detector efficiencies. Putting in the transmission efficiencies of the analyzers and acceptance angle measured by Freedman and Clauser [5], the expected coincidence rate is given by

$$R_{\phi}/R_0 = 0.2512 + 0.2124 \times \cos 2\phi$$

The results of the Freedman-Clauser experiment confirmed this QM calculation within the estimated errors of the experiment for all nine relative analyzer angles considered.

III. MC SIMULATION OF THE GENERIC WAVE FUNCTION COLLAPSE MODEL AND A LOCAL REALISTIC MODEL

If the generic wave function collapse interpretation of QM is correct, a Monte Carlo (MC) simulation of the Freedman-Clauser experiment should also match this QM calculation. To test this hypothesis a MC simulation of the Freedman-Clauser experiment was created using Mathematica. Random polarization angles with a uniform distribution between $-\pi/2$ to $\pi/2$ were generated for the photons using the standard random number generator within Mathematica.

The probability of photon 1 passing analyzer 1, $P(\phi)$ was calculated using Equation 1 with a random polarization for ϕ . A random number, N , between 0 and the maximum probability was then generated and if $N < P(\phi)$, photon 1 was considered to have been transmitted by analyzer 1 (von Neumann's acceptance-rejection technique).

Since photon 1 and photon 2 are entangled, it was assumed, pursuant to the generic wave function collapse interpretation of QM, that the measurement of the polarization of photon 1 collapsed the wave function such that the polarization of photon 2 was the same as the measured polarization angle of photon 1 (i.e. the angle of analyzer 1). The same MC procedure used for photon 1 was then used to determine if photon 2 was transmitted by analyzer 2. If it was, it was counted as a coincident measurement. To account for the effect of the acceptance angle of the detector on the coincidence ratio (R_ϕ/R_0) a multiplicative coefficient, F_2 was applied to the coincidence ratio. F_2 was determined by fitting the generic wave function collapse model MC simulation to the curve defined by the QM calculation.

A MC run consisted of 100,000 simulated photon pairs at each of the nine analyzer angles used in the Freedman-Clauser experiment and 500 separate MC "experiments" were combined (5×10^7 total simulated photon pairs at each angle) to generate the solid black data points shown in Figure 1.

The general shape of the curve closely matches the QM calculation, but with disagreement of approximately 3% (7 to 10 standard deviations) at analyzer angles less than or equal to $\pi/8$ and disagreement between 15% and 52% (18 to 48 standard deviations) for analyzer angles greater than or equal to $3\pi/8$. The only free parameter in the fit was the acceptance angle coefficient, F_2 . The best fit occurred with $F_2 = 0.9222 \pm 0.0002$. This value for F_2 was subsequently used in all other MC simulations to account for the detector acceptance.

Since the QM calculation does not include an explicit assumption regarding the polarizations of the two photons, it is possible the physical correlation between them is either different than assumed here or simply undefined and the generic wave function collapse model should be considered merely an approximation of the underlying physical reality. To address the first possibility several

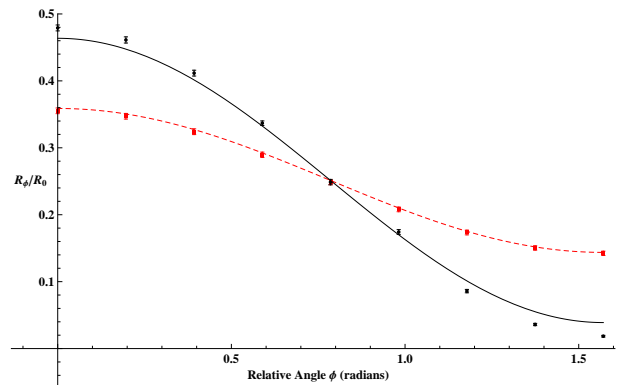


FIG. 1. The solid black line is the QM calculation, black data points are MC simulations assuming the generic wave function collapse model; the dashed red line is the classical calculation assuming a local realistic model, the red data points (squares) are MC simulations of a local realistic model. The error bars represent ± 3 standard deviations.

other physical correlations were tried ranging from no correlation at all to fixed or random differences. No alternative assumption produced better agreement with the QM calculation.

A local realistic model was specifically evaluated which assumed the polarizations of the two photons were random but equal. This physical interpretation is in good agreement with a classical calculation using this assumption (Equation 3 where the variables have the same meanings as in Equation 2) but is in strong disagreement with the results of the Freedman-Clauser experiment and the QM calculation.

$$R_\phi/R_0 = \frac{1}{4}(\epsilon_{\parallel}^1 + \epsilon_{\perp}^1)(\epsilon_{\parallel}^2 + \epsilon_{\perp}^2) + \frac{1}{8}(\epsilon_{\parallel}^1 - \epsilon_{\perp}^1)(\epsilon_{\parallel}^2 - \epsilon_{\perp}^2) \cos 2\phi \quad (3)$$

The red, dashed curve on Figure 1 shows the classical calculation (Equation 3 using the efficiency values obtained by Freedman and Clauser) and the red, square data points show the results of the MC simulation. The same MC procedures described previously were used in this simulation including the same value of the detector acceptance coefficient, F_2 , determined in the generic wave function collapse model fit. Other fixed correlations between the two photons were tried but none produced results that better fit the QM calculation.

IV. MC SIMULATION OF A DYNAMIC STATE REDUCTION/COLLAPSE MODEL (SMEARED POLARIZATION)

We next tested an interpretation of QM wherein physical quantities or "elements of physical reality" using EPR terminology, have no precise values and are fundamentally indeterminate (i.e. smeared). We believe

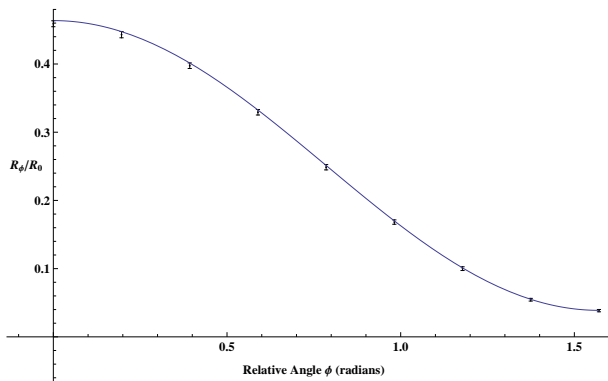


FIG. 2. The solid black line is the QM calculation, black data points are MC simulations based on a smeared polarization model. The error bars represent ± 3 standard deviations.

this situation approximates the physical interpretation represented by dynamical state reduction/collapse models [9, 12, 14, 15].

To explore this possibility a MC simulation was developed wherein neither photon 1 nor photon 2 had precise polarizations. The polarization of photon 1 was again assumed to be random with a uniform distribution from $-\pi/2$ to $\pi/2$. The probability of photon 1 being transmitted by analyzer 1 was calculated using the same technique used in the generic wave function collapse model simulation.

If photon 1 was transmitted by analyzer 1, the polarization for photon 2 was assumed to have a Gaussian distribution about the analyzer 1 angle. The standard deviation of this Gaussian distribution is a function of the presumed inherent uncertainty or “smearing” of the photon’s polarization. To determine if photon 2 was transmitted by analyzer 2, a random number was generated using this Gaussian distribution. This number was taken to be the effective polarization of photon 2. Whether or not photon 2 was transmitted by analyzer 2 was then determined using the same procedure as in photon 1. If both photons were transmitted by their respective an-

alyzers a coincident detection was counted. The same detector acceptance coefficient, F_2 , was applied to the coincidence rate.

The Gaussian standard deviation that produced the best fit to the QM calculation was $\sigma = 0.2131 \pm 0.0009$. Figure 2 shows the results of the MC simulation (data points) and the solid black line is the QM calculation. The smeared polarization model shown in Figure 2 differs from the QM calculation by approximately 1% (1 to 3 standard deviations) for all nine analyzer angles. This difference could be reduced to roughly 0.1% by adjusting the F_2 parameter by 1%. The mean chi squared for the generic wave function collapse model was 0.021 ± 0.002 and 0.0003 ± 0.0002 for the smeared polarization model which is almost a two orders of magnitude better fit without adjusting the F_2 parameter. However, even with no F_2 parameter adjustment the smeared polarization model is a quasi-two variable fit so one would expect it to better match the QM calculation. The amount of improvement, however, at least suggests that a smeared polarization interpretation is a better model of physical reality and appears to support dynamical state reduction/collapse models.

The fit to the Gaussian standard deviation, σ , also appears to support dynamical state reduction/collapse models. Dynamical state reduction/collapse models utilize a “localization distance” [9, 11] or “correlation length (r_c)” [13, 17], generally taken to be on the order of $r_c \simeq 10^{-5}$ cm. Current experiments to test dynamical state reduction/collapse models do not provide a bound on r_c [13].

The atomic cascade of calcium used in the Freedman-Clauser experiment produced photons with an average wavelength of 4.87×10^{-5} cm (5513Å and 4227Å respectively for the two photons). Taking this as the “size” of the photons, an uncertainty in the polarization angle of 0.2131 ± 0.0009 radians equates to a distance uncertainty of $(1.04 \pm 0.14) \times 10^{-5}$ cm in close agreement with the assumed value of r_c used in dynamical state reduction/collapse models.

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